OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find the equation of the tangent to the curve $y = \dfrac{5x+4}{3x-8}$ at the point $(2, -7)$. **[5]**

2 It is given that θ is the acute angle such that $\cot\theta = 4$. Without using a calculator, find the exact value of

(i) $\tan(\theta + 45°)$, **[3]**

(ii) $\operatorname{cosec}\theta$. **[2]**

3 The volume, V cubic metres, of water in a reservoir is given by

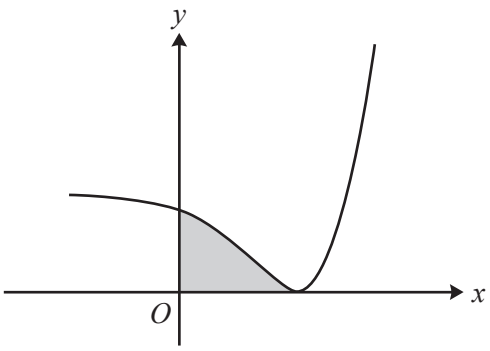
$$V = 3(2 + \sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. **[5]**

4 It is given that $|x + 3a| = 5a$, where a is a positive constant. Find, in terms of a, the possible values of

$$|x + 7a| - |x - 7a|. \quad \textbf{[6]}$$

5

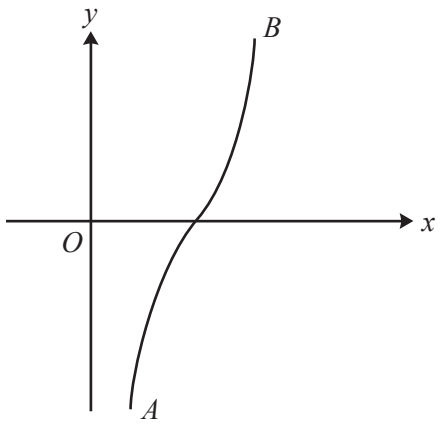


The diagram shows the curve $y = e^{3x} - 6e^{2x} + 32$.

(i) Find the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0. **[4]**

(ii) Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes. **[4]**

6



The diagram shows the curve $y = 8\sin^{-1}\left(x - \frac{3}{2}\right)$. The end-points A and B of the curve have coordinates $(a, -4\pi)$ and $(b, 4\pi)$ respectively.

(i) State the values of a and b. **[2]**

(ii) It is required to find the root of the equation $8\sin^{-1}\left(x - \frac{3}{2}\right) = x$.

(a) Show by calculation that the root lies between 1.7 and 1.8. **[3]**

(b) In order to find the root, the iterative formula

$$x_{n+1} = p + \sin(qx_n),$$

with a suitable starting value, is to be used. Determine the values of the constants p and q and hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. **[5]**

7 **(i)** Find the exact value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$. **[4]**

(ii) Use Simpson's rule with two strips to show that an approximate value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$ can be expressed in the form $m + n\sqrt[3]{36}$, where the values of the constants m and n are to be stated. **[3]**

(iii) Use the results from parts **(i)** and **(ii)** to find an approximate value of $\sqrt[3]{36}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. **[2]**

Question 8 begins on page 4.

8 The functions f and g are defined as follows:

$$\mathrm{f}(x) = 2 + \ln(x+3) \text{ for } x \geqslant 0,$$

$$\mathrm{g}(x) = ax^2 \text{ for all real values of } x, \text{ where } a \text{ is a positive constant.}$$

(i) Given that $\mathrm{gf}(\mathrm{e}^4 - 3) = 9$, find the value of a. **[3]**

(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(iii) Given that $\mathrm{ff}(\mathrm{e}^N - 3) = \ln(53\mathrm{e}^2)$, find the value of N. **[5]**

9 It is given that $\mathrm{f}(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

(i) Show that $\mathrm{f}(\theta) = \cos\theta$. Hence show that

$$\mathrm{f}(4\theta) + 4\mathrm{f}(2\theta) \equiv 8\cos^4\theta - 3.$$ **[6]**

(ii) Hence

(a) determine the greatest and least values of $\dfrac{1}{\mathrm{f}(4\theta) + 4\mathrm{f}(2\theta) + 7}$ as θ varies, **[3]**

(b) solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			Attempt use of quotient rule or, after adjustment, product rule	*M1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x-8$ in denominator; allow M1 for numerator the wrong way round	For product rule attempt, *M1 for $k_1(3x-8)^{-1}+k_2(5x+4)(3x-8)^{-2}$ form and A1 for correct constants 5 and –3;
			Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv	A1	Allow if missing brackets implied by subsequent simplification or calculation	
			Substitute 2 to obtain -13 or equiv	A1		
			Attempt to find equation of tangent	M1	Dep *M; equation of tangent not normal	
			Obtain $y=-13x+19$ or $13x+y-19=0$	A1	Or similarly simplified equiv with 3 non-zero terms	
				[5]		
2	**(i)**		State or imply $\tan\theta=\frac{1}{4}$	B1		Note that both parts are to be answered without calculator so sufficient detail is needed
			State or imply use of $\frac{\tan\theta+1}{1-\tan\theta}$	B1		
			Obtain $\frac{5}{3}$ or $1\frac{2}{3}$ or $\frac{20}{12}$ or exact equiv	B1	But not unsimplified equiv (such as $\frac{5}{4}/\frac{3}{4}$)	
				[3]		
	(ii)		Attempt use of correct relevant identity or of right-angled triangle	M1	Such as $\text{cosec}^2\theta=1+\cot^2\theta$, or $\text{cosec}\,\theta=\frac{1}{\sin\theta}$ with attempt at $\sin\theta$, or use of Pythagoras' theorem in right-angled triangle	
			Obtain $\sqrt{17}$	A1	Final answer $\pm\sqrt{17}$ earns A0	
				[2]		

Question			Answer	Marks	Guidance	
3			Differentiate to obtain $kh^n(2+\sqrt{h})^5$	M1	Any non-zero constants k, n; condone presence of -192 here	
			Obtain $9h^{-\frac{1}{2}}(2+\sqrt{h})^5$ or unsimplified equiv	A1	Without -192 now	
			Divide 150 by their derivative, algebraic or numerical	*M1	Using any recognisable attempt at first derivative	
			Substitute $h=1.4$ and evaluate	M1	Dep *M; assume appropriate substitution if calculation goes wrong	
			Obtain 0.06 or 0.060 or 0.0603	A1	But not greater accuracy in final answer; units not needed unless change made to metres and/or hours	
				[5]		
4			Obtain $2a$ as one value of x	B1		Allow solution leading to $a=\frac{1}{2}x$ (B1) and $a=-\frac{1}{8}x$ (M1A1)
			Attempt to find second value of x	M1	By solving equation with signs of x and $5a$ different, or by squaring both sides and attempting solution of quadratic equation with three terms	If using quadratic formula to solve equation, substitution must be accurate
			Obtain $-8a$	A1	And no other values of x	
			Substitute each of at most two values of x (involving a) leading to one final answer in each case and showing correct application of modulus signs in at least one case	M1		
			Obtain $4a$ as final answer	A1	Obtained correctly from $x=2a$	
			Obtain $-14a$ as final answer	A1	Obtained correctly from $x=-8a$	
				[6]		

Question			Answer	Marks	Guidance	
5	(i)		State first derivative is $3e^{3x} - 12e^{2x}$	B1	Or equiv	
			Equate first derivative to zero and attempt solution of equation of form $k_1e^{3x} - k_2e^{2x} = 0$	M1	At least as far as $e^x = c$; M0 for false method such as $\ln(3e^{3x}) - \ln(12e^{2x}) = 0$	
			Obtain $\ln 4$ or exact equiv and no other	A1	Obtained by legitimate method	
			Substitute $x = \ln 4$ or $e^x = 4$ to confirm $y = 0$	A1	AG; using exact working with all detail present: needs sight of $4^3 - 6 \times 4^2 + 32$ or similar equiv	
				[4]		
	(ii)		Integrate to obtain $k_3e^{3x} + k_4e^{2x} + 32x$	M1	For non-zero constants	
			Obtain $\frac{1}{3}e^{3x} - 3e^{2x} + 32x$ or equiv	A1		
			Apply limits correctly to expression of form $k_3e^{3x} + k_4e^{2x} + 32x$	M1	Using limits 0 and their answer from part (i)	
			Simplify to obtain $32\ln 4 - 24$ or $64\ln 2 - 24$	A1	Or suitably simplified equiv	
				[4]		
6	(i)		State or clearly imply $a = \frac{1}{2}$	B1	$a = \frac{5}{2}$ and $b = \frac{1}{2}$ earn B0 B0	
			State or clearly imply $b = \frac{5}{2}$	B1	$\sin(-\frac{1}{2}\pi) + \frac{3}{2}$ and $\sin(\frac{1}{2}\pi) + \frac{3}{2}$ earn B0 B0	
			(Implied by, for example, just $\frac{1}{2}$ and $\frac{5}{2}$ stated in that order)			
				[2]		

Question			Answer	Marks	Guidance	
	(ii)	(a)	Carry out relevant calculations using radians	M1	Involving $8\sin^{-1}(x-\frac{3}{2})$ or $8\sin^{-1}(x-\frac{3}{2})-x$ or equiv; needs two explicit calculations	May carry out calculations in, for example, $\frac{3}{2}+\sin(\frac{1}{8}x)-x$
			Obtain 1.6 and 2.4 or –0.1 and 0.6	A1	Or equivs	
			Conclude with reference to $1.6<1.7$ but $2.4>1.8$, or to sign change	A1 **[3]**	Or equiv	
		(b)	State or imply $p=\frac{3}{2}$ and $q=\frac{1}{8}$	B1	Implied by presence in iterative formula	
			Obtain correct first iterate	B1	Having started with value x_1 such that $1.7\le x_1\le 1.8$; given to at least 4 s.f.	
			Carry out iteration process	M1	Obtaining at least three iterates in all; having started with any non-negative value; implied by an apparently converging sequence of plausible values; all values to at least 4 s.f.	Answer only can earn no more than the first B1 for values of *p* and *q*; working in degrees can earn no more than the first B1 (for *p* and *q*) and M1
			Obtain at least three correct iterates	A1	Allowing recovery after error	
			Conclude with clear statement that root is 1.712	A1 **[5]**	Final answer required to exactly 4 significant figures	
7	(i)		Integrate to obtain integral of form $k(7x+1)^{\frac{4}{3}}$	*M1	Any non-zero constant *k*	
			Obtain $\frac{3}{28}(7x+1)^{\frac{4}{3}}$	A1	Or unsimplified equiv	
			Apply limits correctly and attempt exact evaluation	M1	Dep *M; substitution of limits to be seen	
			Obtain $\frac{180}{7}$	A1 **[4]**	Or exact equiv such as $\frac{720}{28}$ or $25\frac{5}{7}$	
	(ii)		Attempt expression of form $k(y_0+4y_1+y_2)$	M1	Any constant *k*; attempting exact *y* values corresponding to *x* values 1, 5, 9	Missing brackets which are not implied by subsequent calculation and which lead to $ky_0+4y_1+y_2$ earn M0
			Obtain $\frac{4}{3}(\sqrt[3]{8}+4\times\sqrt[3]{36}+\sqrt[3]{64})$	A1		
			Obtain $8+\frac{16}{3}\sqrt[3]{36}$	A1 **[3]**	No need for *m* and *n* to be stated separately	

Question			Answer	Marks	Guidance	
	(iii)		Equate answers to parts (i) and (ii) and carry out complete correct relevant rearrangement	M1	Provided $\sqrt[3]{36}$ is involved	Correct answer only seen: M1A1 answer only seen: if follows correctly from their parts (i) and (ii): M1A0
			Obtain $\frac{93}{28}$ or $\frac{372}{112}$	A1 **[2]**	Or equiv of requested form	
8	**(i)**		Obtain 6 or $2+4$ at any stage for application of f	B1		
			Attempt composition of functions the right way round	M1		
			Obtain $a=\frac{1}{4}$ or $\frac{9}{36}$ or equiv	A1 **[3]**		
	(ii)		Obtain expression involving e^{y-2} or e^{x-2}	M1		
			Obtain $e^{x-2}-3$	A1		
			State $x \geq 2+\ln 3$ or equiv	B1 **[3]**	Not for >; not for decimal equiv ; using x	
	(iii)		Either: Apply f once to obtain $2+N$	B1		
			Apply f to their expression involving N	M1		
			Obtain $2+\ln(N+5)$ or $2+\ln(2+N+3)$	A1		
			Attempt solution of equation of form $2+\ln(pN+q)=\ln(53e^2)$	M1	Involving manipulation so that value of N is apparent	
			Obtain 48 from correct work	A1		

Question			Answer	Marks	Guidance	
			Or 1:			
			Obtain $\mathrm{ff}(x)$ of form $k_1 + \ln[k_2 + \ln(x+3)]$	M1	Or equiv with immediate substitution for x; missing bracket(s) may be implied by subsequent work	
			Obtain correct $2 + \ln[5 + \ln(x+3)]$	A1		
			Substitute for x to obtain $2 + \ln(N+5)$	A1		
			Attempt solution of equation of form $2 + \ln(pN+q) = \ln(53\mathrm{e}^2)$	M1	Involving manipulation so that value of N is apparent	
			Obtain 48 from correct work	A1		
			Or 2:			
			Apply f^{-1} to obtain $\mathrm{e}^{\ln(53\mathrm{e}^2)-2} - 3$	B1		
			Attempt simplification of expression involving ln and e	M1		
			Obtain $\mathrm{f}(\mathrm{e}^N - 3) = 50$	A1		
			Apply f, or apply f^{-1} to right-hand side	M1		
			Obtain 48	A1		
				[5]		
9	**(i)**		Use at least one addition formula accurately	M1	Without substituting values for $\cos 30°$, etc. yet	
			Obtain $\cos\theta$	A1	AG; necessary detail needed	
			State $\cos 4\theta = 2\cos^2 2\theta - 1$	B1	Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$	
			Attempt correct use of relevant formulae to express in terms of $\cos\theta$	M1	Or in terms of $\cos\theta$ and $\sin\theta$	
			Obtain correct unsimplified expression in terms of $\cos\theta$ only	A1	e.g. $2(2c^2-1)^2 - 1 + 4(2c^2-1)$	
			Simplify to confirm $8\cos^4\theta - 3$	A1	AG; necessary detail needed	
				[6]		

Question			Answer	Marks	Guidance	
	(ii)	(a)	Obtain $\frac{1}{12}$	B1		
			Substitute 0 for $\cos\theta$ in correct expression	M1	No need to specify greatest and least	
			Obtain $\frac{1}{4}$	A1		
				[3]		
		(b)	State or imply $8\cos^4(3\alpha)-3=1$	B1	Or $2\cos^2 6\alpha+4\cos 6\alpha-2=0$	
			Attempt correct method to obtain at least one value of α	M1	Allow for equation of form $\cos^4(3\alpha)=k$ where $0<k<1$ or for three-term quadratic equation in $\cos 6\alpha$	
			Obtain 10.9	A1	Or greater accuracy 10.921…	Answer(s) only: 0/4
			Obtain 49.1	A1	Or greater accuracy 49.078…; and no others between 0 and 60	
				[4]		